IMUB

Universitat de Barcelona

MINI COURSE

HEAT-FLOW MONOTONICITY UNDERLYING SOME SHARP INEQUALITIES IN GEOMETRIC ANALYSIS, HARMONIC ANALYSIS AND DISPERSIVE PDE

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Dates: Wednesdays 5, 12 and 19, December (2012)
Time: 12:00 - 14:00
Lecture room: B2

Course abstract

Abstractly speaking, to prove an inequality $A \leq B$ one may try to locate a quantity $Q(t)$ depending on a "time" variable t such that $Q(t_0) = A$ and $Q(t_1) = B$, and moreover $Q(t)$ is *non-decreasing* for $t \in [t_0, t_1]$.

We will be concerned with inequalities where

$$A = \Lambda(f_1, \ldots, f_m) \qquad \text{and} \qquad B = C(\Lambda) \prod_{j=1}^{m} \|f_j\|_{p_j},$$

where Λ is some functional, $m \geq 1$, each f_j is a function in some Lebesgue space $L^{p_j}(\mathbb{R}^{d_j})$, and $C(\Lambda)$ is some finite constant independent of the input functions f_j.

The quantity Q will arise by taking

$$Q(t) = \Lambda(u_1(t)^{1/p_1}, \ldots, u_m(t)^{1/p_m}),$$

where u_j satisfies the classical heat equation $\partial_t u_j = \Delta u_j$, with initial data $u_j(0) = f_j^{p_j}$. Such an approach is sensible when the optimal constant $C(\Lambda)$ is suspected (or known) to be attained or exhausted by gaussian inputs f_j, in which case t_0 is taken to be zero, and given the large time asymptotics of the solution of the heat equation, t_1 is taken to be infinity. Establishing the monotonicity of $Q(t)$ is typically a rather non-trivial task (when indeed it is true).

During these talks, we will see the above paradigm in the context of the following fundamental inequalities from geometric analysis, harmonic analysis and dispersive PDE:

(1) The Brascamp–Lieb inequality (this includes, for example, the multilinear Hölder inequality and the Loomis–Whitney inequality).
(2) The sharp Young convolution inequality (both forward and reverse forms).
(3) Sharp Strichartz estimates associated to the free Schrödinger propagator.
(4) The sharp Hausdorff–Young inequality for the Fourier transform.
(5) The Bennett–Carbery–Tao multilinear Kakeya inequality.

The Brascamp–Lieb inequality in (1) above is a powerful multilinear inequality and understanding the heat-flow monotonicity phenomena in this setting will be important for understanding such phenomena associated with the inequalities in (2)–(5).

Titles of each talk

(1) An introduction to the Brascamp–Lieb inequality
(2) Heat-flow monotonicity for the Brascamp–Lieb inequality
(3) The sharp forward and reverse Young convolution inequality via heat-flow
(4) Sharp Strichartz estimates for the free Schrödinger equation via heat-flow
(5) The sharp Hausdorff–Young inequality and heat-flow
(6) The Bennett–Carbery–Tao multilinear Kakeya inequality